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SF-8165

B. E. - II (Sem. IV) Examination

May / June – 2011

Engineering Maths : Paper - IV

(Common for All)

(New Syllabus)

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. - 2 (Sem. 4)**

Name of the Subject :

☛ **Engineering Maths - 4**

☛ Subject Code No. : **8 1 6 5** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt **all** questions.

(3) Figures on the right indicate marks.

(a) Do as directed : **10**

(1) Define analytic function.

(2) Write Cauchy-Riemann equations in polar form.

(3) Find the value of $\operatorname{Re}(f(z))$ and $\operatorname{Im}(f(z))$ at the indicated point where $f(z) = \dfrac{1}{1-z}$ at $7+2i$.

(4) If $z = re^{i\theta}$, find $\left|e^{iz}\right|$.

(5) Find all points at which the mapping $w = z^2 + \dfrac{1}{z^2}$ is not conformal.

(b) Solve any **two** : **6**

(1) Find and plot all roots of $\sqrt[3]{8i}$.

(2) If $u = \log\left\{\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)\right\}$ then prove that $\tanh\dfrac{u}{2} = \tan\dfrac{\theta}{2}$.

(3) Determine the analytic function whose imaginary part is $e^{x}(x\cos y - y\sin y)$.

2 (a) State Cauchy-Integral theorem. Evaluate $\oint_C (z^2 - 3z + 2)\, dz$, where C is a circle $|z| = 2$. **5**

(b) Solve any **three** : **12**

(1) Find the image of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Also show the regions graphically in both the planes.

(2) Find the bilinear transformation which maps the points $i, -1, 1$ of the z – plane into the points $0, 1, \infty$ in W-plane respectively.

(3) Evaluate $\int_C \frac{2e^{z}}{(z-a)^3} dz$ where the point lies within the close contour C.

(4) Integrate $f(z) = x^2 - iy$ along the curve $y = x$ from $z = 0$ to $z = 1 + i$.

3 (a) Determine the residue of $\frac{3z-4}{z(z-1)(z-2)}$ at each of its poles in the finite Z – plane. **5**

(b) Solve any **three** : **12**

(1) Determine and classify the singularities of the following functions :

(i) $f(z) = \frac{1}{z - z^3}$ (ii) $f(z) = \frac{e^{z}}{1+z^2}$

(2) Find the Cauchy-principal value of $\int_{-\infty}^{\infty} \frac{dx}{x^2 - ix}$.

(3) Find the Maclaurin's series expansion of $f(z) = \tan^{-1} z$.

(4) Determine Laurent's series expansion of $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $1 < |z| < 3$.

4 (a) Do as directed : **10**

(1) If $f(x)=\frac{1}{x}$, find the divided differences $[a, b]$ and $[a, b, c]$.

(2) Set up a Newton iteration for computing the cube root of a given positive number N.

(3) Find $\Delta \sin x$.

(4) State Simpson's $\frac{1}{3}$ rule.

(5) Obtain the first approximation of the Picard's method for $\frac{dy}{dx}=x-y^2$ at $y(0)=1$ when $x=0.1$.

(b) Attempt any **three** of the following : **12**

(1) Find the polynomial which takes the following values $y(0)=1$, $y(1)=0$, $y(2)=1$ and $y(3)=10$. Also find $y(4)$.

(2) State Trapezoidal rule with $n=10$ and evaluate $\int_0^1 e^{-x^2} dx$.

(3) Evaluate $\int_0^3 \frac{dx}{1+x}$ with $n=6$ using Simpson's $\frac{3}{8}$ rule.

(4) Compute $\int_0^2 (1+x)dx$ by a Gauss two point quatrature.

5 (a) Solve any **two** of the following : **8**

(1) Solve the following system of equations using partial pivoting by Gauss-Elimination method :

$8x_2 + 2x_3 = -7$

$3x_1 + 5x_2 + 2x_3 = 8$

$6x_1 + 2x_2 + 8x_3 = 26$

(2) Solve using Gauss-Jordan method :

$x+2y-z=3$

$3x-y+2z=1$

$2x-2y+3z=2$

(3) If $y(35.0)=1175$, $y(35.5)=1280$, $y(39.5)=2180$, $y(40.5)=2420$, find $y(40)$ by using divided differences.

(b) Attempt any **two** of the following : **6**

(1) Find a real root of the equation $x^2-3x+1=0$ using Bisection method correct upto three decimal places.

(2) Find a root of the equation $x^3-9x+1=0$ using Newton-Raphson method correct upto three decimal places.

(3) Use the Secant method to find the root of $f(x)=e^{-x}-x$. Start with the initial estimates of $x_0=0$ and $x_1=1$.

6 (a) Attempt any **two** of the following : **8**

(1) Use Jacobi's method to solve the equation :

$5x-y-z=3$

$-x+10y-2z=7$

$-x-y+10z=8$

(2) Use Gauss-Seidal method to solve the equation :

$10x-5y-2z=3$

$4x-10y+3z=-3$

$x+6y+10z=-3$

(3) Find both eigen values and eigen vectors of $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ by power method.

(b) Attempt any **two** of the following : **6**

(1) Use Runge-Kutta method of fourth order to calculate $y(0.2)$ given $\frac{dy}{dx}=x+y$, $y(0)=1$ taking $h=0.1$.

(2) Apply improved Euler method to solve the initial value problem $\frac{dy}{dx}=x+y$ with $y(0)=0$ choosing $h=0.2$ and compute y_1, y_2, y_3, y_4, y_5.

(3) Solve $\frac{dy}{dx}=x^2+y^2$ using the Taylor's series method for the initial condition $y(0)=0$ where $0 \le x \le 0.4$ and $h=0.2$.